

In the Steps of the Master Mechanic

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Introduction

I report continuing progress with my work on the Antikythera Mechanism, the oldest known geared device, the fragments of which, recovered from a shipwreck dateable to the first century B.C., are preserved in The National Archaeological Museum in Athens.¹

According to the widely-accepted interpretation of Price, the instrument had dials on two opposite faces, with indicators connected by wheelwork that was probably moved by hand. The "front" display showed the mean places of the Sun and Moon in the Zodiac, and the date. The "back" bore a display showing the age or phase of the Moon and other less well defined but presumably related functions.² Most subsequent accounts are variants based on Price's work.

Recently I outlined a new reconstruction of the front dial, in which the conjectural restoration of epicyclic wheelwork offered a rationale for hitherto unexplained features of the original fragments.³ It led to the possibility of including epicyclic mechanism to modify the Sun's motion according to the solar theory of Hipparchus and to show an approximation to the motion of the inferior planets according to the simple epicyclic theory associated with Apollonius of Perga. The scheme could be extended, beyond but still consistent with the physical evidence, to include the lunar theory of Hipparchus and the motion of the superior planets. The possibility of developing such a display for the Antikythera Mechanism is very suggestive in the light of contemporary literary accounts of planetaria.⁴

I have subsequently built a working model, to the scale, and in the style, of the original, both as an aid to developing this idea and as a way of illustrating its realisation. I have given an illustrated account of this model, in its early form, elsewhere.⁵ Here I describe it in its present more refined form and offer some historical, mathematical and mechanical considerations that lie behind its design.

My work is based on new observations of the original fragments made jointly by A.G. Bromley and myself.⁶ These augment considerably, sometimes call into question, and sometimes contradict the detail of Price's description. In particular, it is certain that the train of gears connecting the large wheel that Price termed the "drive wheel", wheel **B1**, and the central arbor concentric with it, is such that the two turn in the same sense, not in opposite senses as Price states.⁷ It is this observation that makes my reconstruction of the front dial possible.

This revision forces us to reconsider our understanding of the arrangement and function of the rest of the mechanism. However, since the two mobiles mentioned are the only ones that are closely connected to the front dial display, and as I am here concerned only with this display, I defer this discussion to a further paper now in preparation.

Provisionally, however, I accept Price's statement that wheel **B1** and the central arbor turn at rates in the ratio 19:254, the Metonic ratio. That is, the ratio of their periods of rotation approximates to (tropical year):(tropical month). Pointers turning rigidly at these rates would describe simultaneously the motion of the Mean Sun and the Mean Moon through the Zodiac. I term the large wheel the Mean Sun wheel, and the central arbor the Mean Moon arbor.

Restorations to the Mean Sun wheel: Sun, Inferior Planets and Date

The Mean Sun wheel turns at the correct rate to serve as the platform for epicyclic models for the True Sun and inferior planets. The need to carry both the epicycle discs and the trains driving them could offer explanations for its size and for the remains of the structure on it. The presence of a central boss with a squared end also hints at an epicyclic train, driven by running around a stationary wheel mounted on the square.

Hipparchus's solar theory requires the Sun to move with constant angular velocity around a circle that is not centred on the Earth.⁸ It is equivalent to adding a constant vector offset to a central circular motion, which can be effected by having an epicycle that remains fixed in direction. This may be achieved using three wheels. The stationary central wheel and the final wheel fixed to the epicycle have equal numbers of teeth and are connected by an idle wheel.

To obtain adequate approximations to the desired rates of rotation (a point which is discussed below in the section *The Question of Accuracy*), and to keep the numbers of teeth on each wheel within reasonable bounds, compound trains are adopted to drive the planetary epicycles. The numbers chosen are given in table 1. These trains and that for the Sun are contrived to work from a single fixed central wheel, and to have several further wheels in common, as will be seen by comparing the tooth counts in the table.

This wheelwork may be fitted comfortably, occupying much of the available space on the Mean Sun wheel, if gear pitches similar to those found in the original fragments are adopted. It is not suggested, however, that the original Mean Sun wheel must have carried just this arrangement of wheels. It is indeed possible that a simpler arrangement, yielding cruder approximations to the desired ratios, might match the evidence of holes and other fixings now seen.⁹ The point of building the model in its present form was, however, to show that there is no difficulty in devising practicable trains that yield good approximations.

A further explanation for the presence of the large Mean Sun wheel is to be found in the geometry associated with the large epicycle for Venus, which leads to the development of large forces which would be hard to accommodate on a smaller scale.

The date must be indicated by a hand moving around the dial at a constant rate. The True Sun hand does not do so, and a separate date hand is introduced, rotating rigidly with the Mean Sun wheel.

Further Restoration for Internal Consistency: the Moon

There is no physical evidence for any modification of the Moon's mean motion, given by the Mean Moon arbor, but a realisation of Hipparchus's lunar theory is chronologically justifiable and is included for the sake of consistency with the solar model. An epicyclic disc must turn slowly backwards. As with the planetary models, an approximation is entailed in obtaining this rotation, and the solution chosen is discussed below and given in table 1. A compound epicyclic train is driven by running around a second fixed central wheel. It is mounted on a small platform on the central arbor, fitted between the arbors of the epicycle discs for the True Sun, Mercury and Venus.

Further Restoration for Internal Consistency: the Superior Planets

The distinction was made between the inferior and the superior planets simply on account of the difference in their observed behaviour, and neither group seems to have been preferred in antiquity as more important than the other. Therefore if the designer of the Mechanism modelled one group, it seems likely that he would have been interested in modelling both. I show that it is possible to extend the reconstruction to add the superior planets. The degree of complication is not increased significantly, and the whole remains consistent with the evidence of the original fragments.

For the inferior planets the epicycles rotate at individual rates but both are carried around the Earth at the rate of the Mean Sun. For the superior planets, however, the epicycles all rotate at the rate of the Mean Sun and are carried around the Earth at individual rates. Therefore the motions of Mars, Jupiter and Saturn call for three additional platforms, and in the model these are contained within separate stages. Each stage works in the same way, but both the proportions of the epicycle discs and the tooth-counts of the gears differ.

In each stage the first central wheel is driven at the rate of the Mean Sun wheel. Motion at this rate is then transferred to the epicycle disc by a system of two equal wheels connected by an idle wheel on the platform, analogous to that used in modelling Hipparchus's theory of the Sun. In these assemblies, however, the idle wheel also forms part of a compound train leading back to engage a fixed wheel at the centre, which causes the platform itself to rotate.

In the present model the stages are driven from the Mean Sun wheel through a side arbor, but they could instead be driven from the centre. In case the side arbor should be thought objectionable, it is proposed to model this elegant but slightly more complicated alternative later, although the present scheme works extremely well. No refinement of the detail, however, can alter the fact that the entire scheme of wheelwork for the superior planets is, and probably must always remain, conjectural.

The Case

Traces of woodwork found in the original fragment **A** suggest a case of plan dimensions that would enclose the reconstructed planetary mechanism rather neatly, although it would have to be stepped out, like a letter **T**, to accommodate the elongated back dial (which is yet to be added to the model).¹⁰ Because there is no certain join between the original fragments **A**, including the frame plate, and **C** (**Γ**), including the dial, their separation, and so the depth of the case, may be chosen to suit. The reconstructed case is made to hold the frame plate, the dial, and

the stages for the superior planets in correct alignment. The structure is so designed that, left for 2000 years in seawater, it might plausibly degenerate into what now remains of the original.

The Front Dial

Some details of the piece of dial plate preserved in fragment C (Γ) have not previously been noticed or understood. The inner ring, divided into 360° and with the names of the 12 Zodiacal constellations, is engraved directly on the dial plate. The outer ring, divided into 365 days and with the names of the months of the Egyptian calendar, is engraved on a separate ring that is let into a close-fitting groove. This allows the necessary adjustment of the date to astronomical events of one day every four years, which was a feature of this calendar.

In the dial plate, behind the calendar ring, is a ring of holes. Measurements made on a radiograph of the surviving portion suggest that there were 365 holes in the full circle.¹¹ Since there is no evidence for any other means of holding the ring in place, it is suggested that the ring had wire "feet" projecting from its back, passing through some of the holes. (In the model, five equally-spaced feet are fitted.) There remains part of a curved structure that appears to have been fastened to the back of the dial plate, which may be identified tentatively as a spring clip to grip such feet.¹² In any case, fitted with feet that pass through the holes, the ring can be lifted out, turned through any angle corresponding to a whole number of days, and dropped back into the groove.

It is tempting to suppose that the calendar ring might have been moved round automatically, but no trace of mechanism to effect this has been found. Moreover it is hard to see how any such arrangement could be compatible with the evidence that we do have.

On the preserved corner of the dial is a small sliding bolt, worked by a thumb-button on the front. This appears to be intended to hold the dial in. In the model four such bolts are fitted, one to each corner of the dial plate, their tongues working behind pieces of plate fixed to the two ends of the case. Alternatively one edge of the dial might have been held down by a fixed ledge, or might have been fitted with a hinged joint. In any case, the provision of bolts suggests that the user of the instrument needed regular access below the dial, perhaps to reset an approximate astronomical train.

The Question of Accuracy

A measure of the performance of this reconstruction is obtained by supposing the instrument to be set by date, and then considering how well the places of Sun, Moon and planets, as shown on the dial, might correspond with observation or independent computation.

The indications of the Sun and of the date are directly related, and Hipparchus's solar theory is adequate. Therefore, if the instrument is correctly made and adjusted, there is no observable error in the indication of the Sun. The other indications are all subject to two distinct types of error. Long-term cumulative errors are due to the adoption of gear trains that render the intended periods only approximately. Short-term cyclical errors are due to the adoption of inadequate models.

A criterion for the acceptable degree of cumulative error can be obtained by considering the approximation that, as mentioned above, is thought to be embodied in the wheelwork of the original fragment A:

$$19 \text{ tropical years} = 254 \text{ tropical months.}$$

This, the Metonic Ratio, may also be stated thus:

$$19 \text{ years} = 235 \text{ synodic months.}$$

In this form it may be compared with the ratio of parameters attributed to Hipparchus:

$$\begin{aligned} \text{Tropical year} &= 365 + \frac{1}{4} - \frac{1}{300} \text{ days;} \\ 4267 \text{ synodic months} &= 126007 \text{ days, 1 hour.} \end{aligned}$$

The two are found to agree to better than 1 part in 2,500,000. Using both to find the ecliptic longitude of the Mean Moon as a function of time, the discrepancy between them amounts to only 1°, one dial division, in about 520 years.

Comparison of the Metonic ratio with modern data shows it to be in reality a less good approximation to the phenomena, leaving us to wonder about the data from which, according to Ptolemy, Hipparchus derived his parameters. Nevertheless, accepting the close agreement between the Metonic ratio and the Hipparchian parameters as a criterion of adequate long-term performance, it is interesting to show that each of the trains in the reconstructed instrument can be designed to yield a long-term performance at least as good. In each case the long-term error may be kept to less than 1° in 500 years.

For want of strictly contemporary parameters for planetary periods, those of Ptolemy were taken. From these a ratio was derived for each function, and for each a set of vulgar fractions, approximating to the desired ratio, was generated. For each fraction the factors of numerator and denominator were tabulated, together with its degree of approximation to the intended ratio.¹³

The rate at which error accumulates depends on how fast the output runs, and the way in which this accumulated error affects the dial indication depends on the instantaneous geometry of the kinematic model that the train is driving. To take account of the latter effect, "magnification factors" were found corresponding to the most adverse configuration of each model.

From these numbers gear trains of appropriate precision, and which would fit within the instrument, were devised. It was necessary to use compound trains throughout, but all the wheels have numbers of teeth and pitches lying within the range found in the original fragments. The actual tooth-counts are of no importance except as an illustration that the challenging design criterion set out above can actually be met. It is in this spirit that they are presented in table 1.

In practice, the small cumulative errors are often swamped by short-term cyclical errors.

For example, Ptolemy demonstrates that Hipparchus's lunar model yields an error of rather more than $2\frac{1}{2}^\circ$ at dichotomy.

The cyclical errors inherent in the use of simple epicyclic models for the planets can readily be appreciated, except in the case of Mercury, by comparison with Ptolemy's models. There are two causes of discrepancy. Firstly, the eccentricity of the equant circle in the Ptolemaic model gives rise to a corresponding angular displacement of the centre of the epicycle (error A). Secondly, the distance of the epicycle from the Earth varies due to the eccentricity of the deferent circle, and so the observed angular size of the epicycle changes (error B). Both effects give rise to discrepancies that vary sinusoidally with elongation of the centre of the epicycle from apogee, but the first has maxima at $\pm 90^\circ$ while the second has maxima at apogee and perigee. Therefore these maxima, shown in table 2, never occur together.

The complication of the Ptolemaic model for Mercury makes it harder to make a similar comparison in this case. Besides, it is well known that this model is rather unsatisfactory, and that the closeness of Mercury to the Sun makes observation of it difficult. Mercury's greatest elongation actually varies from about 19° to $26\frac{1}{2}^\circ$. The simple epicyclic model, with its fixed greatest elongation, is therefore bound to show an occasional readily-observable maximum error of at least $3\frac{3}{4}^\circ$ in this respect alone. Without going into the matter in more detail, one may perhaps treat this figure as a rough indication.

These cyclical errors can be reduced only by modifying the models: that is, by making them more complex. That was to be Ptolemy's achievement, and he seems to claim that it was all his own work, while suggesting that Hipparchus left no planetary theory because he could devise none that would satisfy him. Probably, therefore, it would be inappropriate to attempt to model any elaboration of the simple epicyclic theory in reconstructing the Antikythera Mechanism. In assessing how plausible this reconstruction may be, what really matters is how far the errors of the instrument could have been detected, and how far they might have been held to matter. The latter question is hard for us to answer. Ptolemy's planetary models, in their turn, do not "save the phenomena" uniformly well, and yet they continued to be accepted for one and a half thousand years.

Historical-Mechanical Considerations

Each astronomical indication is governed by an epicycle, represented by a disc on which an eccentric pin indicates the angular position of the body. Each disc turns on its axis, driven by epicyclic gearing on the rotating platform that carries it. The position of each pin is picked up by a slotted radial arm that, turning about the centre on a pipe, transfers the motion up to a hand at the level of the dial.

A precedent for the use of epicyclic gearing is to be found in the differential gear within the original fragment **A** itself.¹⁴ For the rest, the overall arrangement may seem obvious enough, but two elements should not be accepted without question: the use of the pin and slotted arm, and the use of concentric pipes.

The ensemble of pin and slotted lever has been widely used by the clockmaker in more or less detailed models of the Ptolemaic system at least since the late middle ages, but it cannot

be traced back to antiquity.¹⁵ The pin is, in effect, a crankpin, and we lack evidence that the crank was known earlier than the 9th. century.¹⁶ Its combination with a slotted lever is first described in a Persian book of the early 13th. century where it is used to convert rotary to reciprocating motion, most prominently in driving force pumps from a waterwheel.¹⁷ Just possibly it may be significant that, like much else in the Arab technical repertoire, the force pump was a Hellenistic device.¹⁸ While the evidence from antiquity suggests that pumps were usually worked by a rocking hand-lever, it is conceivable that the pin-and-slotted-lever driving gear was also ancient.

The dial rings of large diameter suggest the concentric display that has been adopted. It is possible that a lost dial centre could have contained subsidiary dials, similar to those found on the back face, on which the places of the planets might have been shown separately. The surviving piece of the front dial, however, seems to have a clean edge at the inside of the zodiacal ring, with no sign of having had a dial centre attached to it.¹⁹ Advantage is taken of the open centre to place some of the hands below the plane of the dial, so as to reduce the height, and hence the parallax, of the uppermost hands above it.

A display with subsidiary dials could be served by a simpler mechanical arrangement than that needed for a concentric display, but the successful realisation of the more complicated arrangement is still consistent with the evidence.

The lowermost hand, for Saturn, is worked by the uppermost stage of the mechanism and is driven directly by its pin. The uppermost hand, for the Moon, is fitted to the squared end of the central arbor. Each other hand is mounted on the upper end of one of a concentric array of pipes, each of which has a slotted radial arm engaging the corresponding epicyclic pin at its lower end. Fitting one within another, the pipes support each other but turn independently.²⁰ This arrangement has also been used by the clockmaker since the middle ages, but again we must consider the question of historical precedents.

In fragment **A** of the original Mechanism there are two places in which there are several components turning at different rates on a common axis: probably three each on Price's axis **B**, the centre of the dial system now under discussion, and on axis **E**, on which the differential gear is centred, if we include stationary elements as components turning at zero rates. The germ of the idea of multiple concentric arbors is therefore present within the surviving fragments themselves. In both cases the "pipes" are, apparently, stout and very short, but their existence shows that there is no conceptual difficulty in the use of hollow arbors.

It remains to be demonstrated that the workman of the time could have made the longer, relatively narrow-walled pipes, required for the present reconstruction. Similar metal tubes, fitted to one another, are found in surviving fragments of the musical wind instrument, the *aulos*, in which rotating sleeves of bronze tube are often seen fitted over a structural tube.²¹ A workman who could make tubes with the close fit required for this application would have had no difficulty with the less exacting task of making them with a running fit.

Conclusion

In this partial reconstruction of the Antikythera Mechanism, eight hands move around

the front dial. Seven show ecliptic longitudes for the Sun, the Moon, Mercury, Venus, Mars, Jupiter and Saturn, and the eighth indicates the date. Five epicyclic gear trains are introduced, and, in the present state of the model illustrating this reconstruction, these together contain 41 wheels.

The actual arrangement adopted is largely conjectural. Nevertheless, the inclusion of the epicyclic motions for the Sun, Mercury and Venus, all planted on the Mean Sun wheel, makes sense of the size of that wheel and of the squared boss at its centre. The inclusion of the epicyclic motions for the Sun and the Moon, modelling the theory of Hipparchus, is chronologically plausible. Some slight historical justification for the simple epicyclic models for the planets can be found in the theorem of Apollonius of Perga. There is no direct evidence in the surviving fragments for the inclusion either of the epicyclic motion for the Moon, or for the motion of Mars, Jupiter and Saturn in any form, but these elements complement the others. The whole corresponds with contemporary literary accounts of planetaria. Moreover, this elaborate display is more consistent with the sophistication evident throughout the surviving fragments than is the naïveté of an arrangement that shows merely the mean places of the Sun and the Moon. Taking the points together, this reconstruction claims attention as an essay in what is possible.

It is worth stating the practical point that the reconstruction is not appreciably more complicated than the original. It is merely more extensive. Moreover, no part of the model was more difficult to make than some of the components copied from the original fragments. The whole instrument might be built by a skilled workman using little more than a handful of simple tools.

Whether such an instrument would have been seen as useful seems at present to be a matter for conjecture. Its potential as an aid to the casting of horoscopes need not be taken too seriously but should perhaps not be overlooked.²² In any case, utility may not be an appropriate consideration; literary references suggest that planetaria were appreciated as curiosities, just as they are today.

I have now transferred my attention to reconsideration of the remainder of the instrument. Both the reconstruction offered here and the model illustrating it will, in due course, be extended accordingly. It must, however, be understood that many problems, to some of which attention has previously been drawn, remain to be resolved before this work can in any sense be completed.

My research into the Antikythera Mechanism has been well served by plain and tomographic radiographs made by Bromley and myself. New images, now being prepared by computer processing of these radiographs, should carry my work even further and will certainly facilitate illustration of the evidence on which it is based. Thus, the technology of the Modern World is illuminating the technical achievement of Ancient Greece.

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Table 1: Train Counts

(A distinction is made here between the case in which a wheel *drives* a smaller one, and that in which a wheel *leads* a larger one. The figures are set out in three columns. In each case, the ratio of the products of the figures in the first and third columns yields the overall ratio of the train.)

Moon

desired ratio 0.991545527 approximation 469/473
 acceptable error 1 in 1,600 actual error 1 in 453,000

fixed central wheel	67		
intermediate arbor			
driven		22	
leading	14		
epicycle wheel			43

Sun

model is exact

fixed central wheel	40		
intermediate (idle) wheel		41	
epicycle wheel			40

Mercury

desired ratio 3.151976305 approximation 1680/533
 acceptable error 1 in 340,000 actual error 1 in 498,000

fixed central wheel	40		
first intermediate arbor			
led		41	
driving	60		
second intermediate arbor			
driven		45	
driving	63		
epicycle wheel			26

Venus

desired ratio 0.625493777 approximation 795/1271

acceptable error 1 in 288,000		actual error 1 in 307,000
fixed central wheel	40	
first intermediate arbor		
led		41
driving	60	
second intermediate arbor		
driven		53
leading	62	
epicycle wheel		80

Train transferring motion to superior planet sub-assemblies

Mean Sun wheel	225	
side arbor (all wheels)		45
driven wheels in superior planet sub-assemblies (all alike)		225
(overall ratio is unity)		

Mars

desired ratio 0.880769036		approximation 820/931
acceptable error 1 in 95,000		actual error 1 in 204,000
central driving wheel	49	
first intermediate arbor		
driven		41
leading	38	
second intermediate arbor		
idle wheel		40
central fixed wheel		40
epicycle wheel		49

Jupiter

desired ratio 10.85763220		approximation 1683/155
acceptable error 1 in 15,000		actual error 1 in 25,000
central driving wheel	48	
first intermediate arbor		
led		81
leading	30	
second intermediate arbor		
led		68
leading	31	
central fixed wheel		88

epicycle wheel 48

Saturn

desired ratio 28.43201942 approximation 1450/51
 acceptable error 1 in 6,000 actual error 1 in 44,000

central driving wheel	51	
first intermediate arbor		
led		75
leading	16	
second intermediate arbor		
led		64
leading	18	
central fixed wheel		87
epicycle wheel		51

Table 2: Approximate maxima of cyclical errors

(See text for the distinction between error A and error B, and the meaning of the figure given for Mercury.)

	error A (at $\pm 90^\circ$ from apogee)	error B (at apogee and perigee)
Venus	$2\frac{1}{2}^\circ$	1°
Mars	$11\frac{1}{2}^\circ$	$3\frac{3}{4}^\circ$
Jupiter	$5\frac{1}{4}^\circ$	$\frac{1}{2}^\circ$
Saturn	$6\frac{1}{2}^\circ$	$\frac{1}{2}^\circ$

Moon	$2\frac{1}{2}^\circ$	
Mercury		at least $3\frac{3}{4}^\circ$

Notes.

1. Inventory Number X.15087.
2. D.J. de S. Price: "Gears from the Greeks", *Transactions of the American Philosophical Society*, Vol. 64 No.7, 1974. Reprinted as an independent monograph, Science History Publications, New York 1975. Also published in Greek by Technical Museum of Thessaloniki, 1995. Price drew attention to the clear evidence that the front dial had a calendrical function, calling the instrument a "calendar computer". It is, however, clear that this cannot have been the Mechanism's only function.

3. M.T. Wright & A.G. Bromley, "Towards a New Reconstruction of the Antikythera Mechanism", proc. conference *Extraordinary Machines and Structures in Antiquity*, Ancient Olympia (August 2001), forthcoming.
4. The literary evidence for planetaria in antiquity is well set out by Price (note 2).
5. M.T. Wright, "A Planetarium Display for the Antikythera Mechanism", *Horological Journal*, vol. 144 no. 5 (May 2002), pp. 169-173, and vol. 144 no.6 (June 2002), p. 193.
6. A.G. Bromley, "Observations of the Antikythera Mechanism", *Antiquarian Horology*, Vol.18, No.6, Summer 1990, pp.641-652.

M.T. Wright, A.G. Bromley & H. Magou, "Simple X-Ray Tomography and the Antikythera Mechanism", *PACT* 45, 1995.

A fuller account of our original observations is in preparation.

7. M.T. Wright & A.G. Bromley, "Current Work on the Antikythera Mechanism", proc. conference *Ancient Greek Technology*, Thessaloniki 1997. The revision may be stated in the notation of Price (note 2) and may be followed by reference to his figure 33. Wheels **B4** and **E2i** are present, close to the "base plate" and engaged as Price indicates. Wheel **D2** lies further from the base plate. It does not engage wheel **B4** but engages instead a further wheel of 32 teeth on axis **E** that does not figure in Price's scheme which, I suppose, must be fastened to **E2i**. Any reader, attempting to follow this alteration in Price's plan views (figs. 29 to 31), should be aware that these are in some respects seriously at variance with reality.
8. This and all further astronomical matters in this paper are taken from *The Almagest* (properly Μαθηματικὴ Σύνταξις, *Mathematike Syntaxis*) of Claudius Ptolemy. I have referred to the translation by G.J. Toomer, *Ptolemy's Almagest*, London, Duckworth, 1984, and to O. Pedersen *A Survey of the Almagest*, Odense University Press, 1974.
9. If one were limited to planting the arbors of this wheelwork only on the arms of the Mean Sun wheel, one would probably be obliged to accept simple gear-pairs in place of the compound trains. However, the evidence for holes and other fixings on the arms is found, on close inspection, to be quite equivocal. The attempt to match the restored wheelwork closely to it is therefore deferred until the enhanced images now being prepared are available.
10. Some traces may be seen by direct inspection, but they show more clearly in early photographs such as those reproduced in I.N. Σβορώνος, *Το ἐν Αθήναις Ἐθνικὸν Μουσεῖον*, Athens, 1903 (also published in German: J.N. Svoronos, *Das Athener Nationalmuseum*, Athens, 1908). Further detail is seen in radiographs.
11. Some accounts suggest that these holes may be seen. This is not so; their presence has been detected only through radiography, and their position in depth confirmed by tomography. From the front they are covered by the calendar ring, while from the rear they are covered by the further detail mentioned in this paragraph.

12. No attempt has been made to model this latter feature because it has not been sufficiently understood.

13. I thank Professor A.G. Bromley for having found the approximating vulgar fractions, and the factors of their numerators and denominators, using a computer program of his own devising.

14. See Price (note 2). Strictly, not enough of Price's "differential gear" survives for his interpretation to be quite secure, but the arrangement was undoubtedly epicyclic.

15. This application is found in the mediaeval *planetary equitorium*, in which the epicycle disc is set by hand. Price points out that this instrument cannot be traced with certainty prior to the 11th. century. See D.J. de S. Price, *The Equitorie of the Planetis*, Cambridge University Press, 1955. The first known automated display using the arrangement is that designed and built between 1348 and 1364 by de'Dondi, who however used it in the context of considerable further complication in order to model the Ptolemaic system closely. See G.H. Baillie, H.A. Lloyd & F.A.B Ward, *The Planetarium of Giovanni de'Dondi*, London, The Antiquarian Horological Society, 1974.

16. An illustration of a grindstone turned by a hand winch is found on fol. 35^v of the "Utrecht Psalter", Utrecht University Library MS. 488, which is dated to the 9th. century.

The crank is found in some reconstructions of devices in the works of Hero, and Price (note 2) suggested that the Antikythera Mechanism might have been worked by a "crank handle", but in each case there is no sound basis for this detail.

17. Ibn al-Razzaz al-Jazari, *The book of Knowledge of Ingenious Mechanical Devices*, translated and annotated by D.R. Hill, Dordrecht, Reidel, 1974. The book was written in 1204 or 1206.

18. J.P. Oleson, *Greek and Roman Mechanical Water-Lifting Devices*, Toronto & London, University of Toronto Press, 1984.

19. Displays in which the place of each planet was displayed on its own subsidiary dial, within the centre of a larger dial, seem to have been a feature of some late mediaeval clocks. See, for example: Antonio Simoni, 'La Sfera dei Pianeti', *La Clessidra* 27, 7 (1971). His figure 3 illustrates the arrangement of the dial of a clock by della Volpaia. M^r J. H. Leopold has suggested (private communication) that such displays might have been based on an ancient tradition.

20. It is necessary to provide a means of taking the hands off for disassembly of the mechanism. Several schemes have been tried. In the present arrangement, the pipes are divided. Each slotted bar and each hand is each soldered to a length of pipe. When assembled, the pairs meet at a stepped joint so that the lower part drives the upper part.

21. Commonly the aulos was made of lengths of hollow bone, connected by a sleeving of thin-walled bronze tube. Following an improvement ascribed to Pronomus (ca. 400 B.C.), it was

often equipped to play several different scales by being provided with extra side-holes. The outer sleeves had holes corresponding to those in the body of the instrument, and were designed to be rotated to open or close those holes not covered by fingers. For the aulos to sound properly, the fit of the outer sleeves would have to have been practically airtight, and the appearance of surviving specimens bears this out. I thank D^r M.A. Byrne, whose study on the ancient aulos is in preparation, for these details. (private communication)

22. For information concerning horoscopes I have referred to A. Jones, *Astronomical Papyri from Oxyrhynchus*, Philadelphia, American Philosophical Society (*Memoirs* vol. 233), 1999. The popularity of the horoscope seems to have been increasing in the first century B.C., at least in Oxyrhynchus. In casting a horoscope, the places of Sun, Moon and all five planets were tabulated for the moment of the subject's birth, although often this was done only approximately. The relationship of the Ecliptic to the horizon at the time and place of birth was also required.

The instrument could be extended to yield this further information, by adding a rotating element over the Zodiac ring to represent the horizon. In effect, this entails developing a representation similar to the stereographic projection found on a planispheric astrolabe. It is probable that such an arrangement would be anachronistic and it seems far-fetched to imagine its diurnal motion being automated. Besides, this information may readily be got by simpler means.